# On sorting transmission demands in Elastic Optical Networks with Spatial-Division Multiplexing 

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#### Abstract

In this paper, we analyze the impact of sorting the network users when attending their demands on network performance. For this purpose, we solve Routing, Modulation Level, Spatial mode, and Spectrum Assignment (RMLSSA) problem for elastic optical networks (EONs) with spatial-division multiplexing (SDM) with static network operation. We use several different criteria to order the users to find the best ones in terms of network capacity and spectrum and spatial fragmentation on the network. Experimental results that sorting the users in a descending order obtains better results, notwithstanding the criteria chosen. In these experiments, sorting the users according to their path length (in terms of distance or number of links) achieve better results. Despite the sorting criteria, we found that good practices are to attend first the users with significant resource demands and use the highly efficient user demands to fill the gaps left by the previous ones.


Index Terms-Simulation, Elastic Optical Networks, SpatialDivision Multiplexing, Resource Assignment.

## I. Introduction

The overall Internet traffic sustained growth due to the continuously increasing popularity of various network services and applications [1]. Nowadays, optical networks are the only infrastructure capable of supporting this increasing amount of traffic. However, researchers have found that optical fiber capacity is not limitless, and it can be reached soon. This situation is known as "capacity crunch" in optical communications [2]. The previous analysis encourages researches to evolve current optical network infrastructure.

Several possible solutions have emerged to develop new network architectures. The first course of action is to administrate current network resources efficiently, and the second one is to find smart ways to multiply said resources.

Elastic Optical Networks (EON) has emerged as a new spectrum usage scheme to manage the fiber frequency spectrum efficiently [3]-[5]. This efficiency is possible since the frequency spectrum is divided into small, fixed bandwidths called frequency slot units (FSUs), which can be grouped to satisfy the bandwidth required by each user adaptively.

On the resource multiplication point of view, the current single-mode fiber can be replaced by a multi-core fiber (MCF), using Spatial-Division Multiplexing (SDM) technologies. SDM is a forthcoming optical network technology going beyond the capabilities of WDM/EON systems by enabling parallel transmission of several co-propagating spatial modes [6]-[8]. SDM operates with suitably designed fibers. The modes are guided either through multiple cores placed within a single fiber cladding or in a single fiber core of enlarged dimension and modified refractive index profile.

Both, Elastic Optical Networks and SDM technologies offer complementary solutions to the problem of "capacity crunch" in optical communications. Therefore, their joint operation has been considered to bring numerous benefits. However, these joint technologies introduce new problems to be solved in designing optical networks.

One of the main tasks to be solved by network operators is finding a path to each network connection, and a portion of the spectrum frequencies on the route, measured as numbers of FSUs, problem known as routing and spectrum allocation (RSA) problem. The RSA solutions must satisfy two constraints. First, the same FSU allocated to a specific demand must be available in all links in the user path, denoted as the continuity constraint. Second, in case of demands greater than one FSU, the spectrum assigned must be consecutive (contiguous). This requirement is known as the contiguity constraint.

The RSA problem is one of the most common research problems of EONs. However, choosing a modulation format for each connection is relevant, particularly for long-distance optical communications. For instance, a complicated modulation level allows to efficiently transmit higher bit-rates using less bandwidth than simpler modulation formats, but with a limited optical reach (in kilometers). Therefore, finding a balance between spectral efficiency and optical reach must be considered. The introduction of the spatial dimension by SDM
technologies allows the distribution of the users' demands not only in the frequency domain but also in distributing them over different spatial modes. Therefore, the RSA problem converts into the routing, modulation level, spatial mode, and spectrum allocation (abbreviated as RMLSSA) problem. Consequently, in this work, we focus on solving the RMLSSA problem.

The RMLSSA is considered an NP-complete problem [9][11]. Consequently, its solution tends to be divided into parts, decomposed into the routing sub-problem and the spatial mode and spectrum allocation sub-problem, both separately solved [1], [12], [13]. As a consequence of any solution to these problems, there is a chance that unallocated FSUs remain in the frequency spectrum middle section. This problem is known as fragmentation. The said phenomenon is significant since it can produce a meaningful waste of bandwidth if not adequately controlled. Therefore, the goal is to maximize the spectrum usage or to accommodate as many users as possible on a limited network capacity to decrease the spectrum fragmentation [14], [15].

The RMLSSA solutions focus on improving the routing process ( R ) and the spectrum and spatial assignment process (SSA). However, in several papers, [16]-[18], the order in which users are served is considered significant when the network operates statically (the resources are assigned to the users permanently) because it can directly impact the fragmentation of the spectrum and its available use.

In this work, we solve the RMLSSA problem for elastic optical networks with spatial-division multiplexing technologies with static network operation. We consider different criteria to sort the users before the SSA strategy applying mixed approaches based on the users' bandwidth demands and route lengths, in order to minimize the network fragmentation. We analyzed the sorting decision in terms of spectrum and spatial fragmentation, bandwidth usage, and modulation distribution on the fiber cores. Last, we consider a physical-layer impairments model to consider linear and non-linear signals degradation affecting end-to-end optical communications for a proper modulation format selection.

The remainder of this paper is as follows: Section II presents state of the art on RMLSSA strategies. Next, Section III explains our RMLSSA solution. Section IV illustrates numerical examples of our proposal. And finally, we give some conclusions and remarks in Section V.

## II. State of the Art

The RMLSSA problem must be solved efficiently to improve the spectrum usage on the network. We focused our work on the current static network operation. In a static operation, the routes and spectrum assignments are chosen to operate permanently, seeking to minimize the network capacity and fragmentation. One way is to solve it through optimization techniques.

A common approach to modeling and solving optimization problems in communication networks are mathematical programming (MP) strategies, such as mixed-integer pro-
gramming (MIP) or integer linear programming (ILP) problems [15], [19], [20].

However, these models have a massive amount of variables, and consequently, prohibited execution times, even for small networks. Therefore, they cannot solve them in a reasonable time, for the architectures found in practice [1]. For these reasons, it is often used to solve small network examples [11], such as ring networks of up to 9 nodes.

On the contrary, heuristic and meta-heuristic methods represent a set of approximate optimization techniques that can solve various significant scale problems relatively faster. However, these solutions do not have optimality guarantees, which is only ensured in exact algorithms. Standard algorithms implement $a d$-hoc approaches, in which decisions are made in a single run of the algorithm, e.g., the demands are processed in a particular sequence and are allocated one by one in the network according to some spectrum and routes assignment approach. Then, the FSUs on each link are assigned to the network users satisfying the continuity and contiguity restrictions [21].

Many of surveyed works make use of a variation of RMLSSA algorithm where demands can be sorted according to a specific metric, for instance, in descending order of their shortest-path transmission distance [16], or in descending order of the required number of frequency slots [17], [18], [22]. This order can be applied before solving the frequency spectrum and spatial assignment.

The spectrum and spatial allocation (SSA) techniques found in the literature are Most-Used (MU), First-Fit (FF), Best-Fit (BF) among several variations [20]. In [19] is shown that most approaches use the First-Fit scheme. In this scheme, the FSUs are considered as a sequence. Then, the search for available slots starts on the first FSU in the sequence. The request is accepted if the required number of the same contiguous slots (FSUs) are available on all the links belonging to the user path and the same fiber core. Otherwise, the same request is sent to the next slots on the sequence. If a feasible communication cannot be found, then the communication to this user is not possible [23].

## III. Simulation Strategy

This section comprises the main contribution of the article. First, the model used and the assumptions made are detailed. Then, the simulation strategy to solve the RMLSSA problem is explained.

## A. Physical layer impairments of the optical route

The quality-of-transmission (QoT) of an optical route depends on the accumulation of physical impairments, such as attenuation, ASE noise, dispersion, crosstalk, and nonlinear impairments. We use MCF with single-mode cores, and further, assume a core-to-core distance enough to avoid intercore crosstalk and no coupling among modes.

In the solution of the RMLSSA problem, it is essential to consider the effect of optical route length and modulation level interaction on the QoT. A more significant number of bits per
symbol increases the transmission sensitivity to degradation, making the transmission reach shorter for higher modulation levels [24]. To consider this route length - modulation level constraint, the most common approach is to relate to any modulation format available at the transponder with its maximum transmission reach [14]. Modulation formats used in this work are binary phase-shift keying (BPSK), quadrature phase-shift keying (QPSK), and $\Lambda$-quadrature amplitude modulation ( $\Lambda$ QAM), where $\Lambda$ takes values $8,16,32$, and 64 . Table $I$ is based on [25], and shows the maximum achievable reach (MAR), using single-polarization, as a function of the modulation format and bit rates available at the transponders.

TABLE I
Spectrum requirements in terms of FSUs and Maximum ACHIEVABLE REACH (MAR) FOR EACH BIT-RATE AND MODULATION FORMAT PAIR.

| Modulation | $\mathbf{1 0}$ | $\mathbf{4 0}$ | $\mathbf{1 0 0}$ | $\mathbf{4 0 0}$ | $\mathbf{1 0 0 0}$ | MAR [km] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BPSK | 1 | 4 | 8 | 32 | 80 | 4000 |
| QPSK | 1 | 2 | 4 | 16 | 40 | 2000 |
| 8-QAM | 1 | 2 | 3 | 11 | 27 | 1000 |
| 16-QAM | 1 | 1 | 2 | 8 | 20 | 500 |
| 32-QAM | 1 | 1 | 2 | 7 | 16 | 250 |
| 64-QAM | 1 | 1 | 2 | 6 | 14 | 125 |

## B. Model Description

The network is represented by a graph $\mathcal{G}=(\mathcal{N}, \mathcal{E}, \mathcal{K})$, where $\mathcal{N}$ is the set of nodes in the network, $\mathcal{E}$ the set of unidirectional links and $\mathcal{K}$ is the set of cores in all the network links, with cardinality $N, E$ y $K$ respectively.

The set of users $\mathcal{U}$ with cardinality $U$, is composed by all source-destination node pairs on the graph $\mathcal{G}$ requesting communication between them. Each element $u$ has the form $\left\langle s_{u}, d_{u}, b_{u}\right\rangle$, where $s_{u}$ is the source node, $d_{u}$ the destination node, and $b_{u}$ the bit-rate associated to demand $u$.

User $u$ transmission follows a given route $r_{u}$ between its source and destination nodes. Let $\mathcal{R}=\left\{r_{u} \mid u \in \mathcal{U}\right\}$ be the set composed by all the network users paths. These routes are fixed and can be computed by any algorithm available in the literature [23], [26]-[28]. Also, let the set $\mathcal{L}=\left\{\ell_{u} \mid u \in \mathcal{U}\right\}$ be composed by all users' path length (in kilometers) from source node $s_{u}$ to destination node $d_{u}$.

Let $\mathcal{F}=\left\{f_{u} \mid u \in \mathcal{U}\right\}$ be the set of FSU demanded by user $u$, and the set $\mathcal{M}=\left\{m_{u} \mid u \in \mathcal{U}\right\}$ is composed by the modulation format used by each demand $u$.

Let $\mathcal{C}=\left\{c_{e} \mid e \in \mathcal{E}\right\}$ be the set storing the capacity of all network links, with $c_{e}$ as the total capacity in link $e$ in number of FSU . The $c_{e}$ value is calculated by $c_{e}=\sum_{\forall \mathcal{K}} c_{e k}$, where $c_{e k}$ is the total amount of FSU on the $k$-th core.

Besides, let us define the effective link capacity $\hat{c}_{e}$ as the link capacity genuinely used on link $e$, and the effective core capacity $\hat{c}_{e k}$ as the last FSU used on the $k$-th core in link $e$. To compute the $\hat{c}_{e}$ value, first, we must recognize the last
fiber core used $\hat{k}$ on the link. Finally, we add the effective core capacity of this $\hat{k}\left(\hat{c}_{e \hat{k}}\right)$, with the capacity of all the fiber cores previous to the core $\hat{k}$. Consequently, we compute $\hat{c}_{e}$ as follows:

$$
\begin{equation*}
\hat{c}_{e}=\sum_{k=1}^{\hat{k}-1} c_{e k}+\hat{c}_{e \hat{k}} \tag{1}
\end{equation*}
$$

## C. Proposed Algorithm

Figure 1 illustrates the complete RMLSSA strategy. The inputs are the set of Users $\mathcal{U}$, the network graph $\mathcal{G}$ and the total capacity of the network $\mathcal{C}$, they can also be used as inputs in each of the sub-stages that make up the block diagram of the proposed algorithm.

First, the routes $r_{u}$ for each network user are computed by any method found in literature and stored in $\mathcal{R}$.

Then, the path length $\ell_{u}$ is obtained based on $r_{u}$, allowing to calculate the best modulation format possible $m_{u}$ using Table I. Likewise, Table I outputs the number of FSU needed $f_{u}$ to attend each user according to their bit-rate and chosen modulation format. In this way, the modulation $\mathcal{M}$ sets and the demands in terms of FSU $\mathcal{F}$ are obtained.

Next, the users $\mathcal{U}$ are ordered according to a certain rule in order to minimize the network fragmentation. This step will be explained in detail later. In this step, the output is the same set of users $\mathcal{U}$, but sorted with the chosen criterion. Finally, the spectral and spatial allocation (SSA) procedure is executed for each user $\mathcal{U}$.

The outputs of the RMLSSA proposal are the users path $\mathcal{R}$, the modulation formats used by these users $\mathcal{M}$, and the FSUs and core used by each user stored in the sets $\mathcal{U}$ and $\mathcal{E}$. The proposal was separated into several sub-sections to be further explained in-depth as follows.

1) Routing strategy: A routing strategy consists of finding an optical path between the source and destination nodes for each user on the network. Our proposal uses the shortest path criterion to compute the route $r_{u}$ for each user, using Dijkstra algorithm [27], In algorithmic form, we symbolically write $\mathcal{R}:=\operatorname{Routing}(\mathcal{G}, \mathcal{U})$.
2) Modulation Format and number of FSU: In this section, the modulation format is chosen for each user, based on the bit-rate demands $b_{u}$, and the route length $\ell_{u}$ (in kilometers).

For all users, we obtain the route lengths $\ell_{u}$ according to the users' path $r_{u}$. Then, the most efficient modulation format $m_{u}$ is chosen from Table $I$, that is, the one that requires the least amount of FSU according to the bit-rate demanded $b_{u}$.

For example, for a distance $\ell_{u}=1350[\mathrm{~km}]$, the row associated with $2000[\mathrm{~km}]$ is selected, for which it corresponds to a QPSK modulation. In this way, we obtain the most efficient modulation format $m_{u}$ for each calculated distance $\ell_{u}$.

Similarly, from Table I, we can get the number of FSU $f_{u}$ demanded by the bit-rate - modulation pair. For the same example, a user with $Q P S K$ modulation assignment will need 4 FSUs for 100 Mbps bit-rate.

Symbolically, let us write $\{\mathcal{M}, \mathcal{F}\}:=M L(\mathcal{L}, \mathcal{U})$.
3) Sorting the users: In a static network operation, the users can be sorted previous to the spectrum and spatial assignment


Fig. 1. Diagram showing the inputs required to run our proposal, the necessary steps to perform the method, and the outputs delivered
procedure. In [11], the authors analyzed the impact of choosing an order for the users to be assigned in EON ring topologies since it influences network performance in terms of network capacity and spectrum fragmentation. Consequently, to extend this analysis for EON-SDM architectures, the following sorting criteria are tested in our work:

- Random: The given order does not have any particular criterion, obtaining then a list composed randomly with uniform distribution.
- FSU: The order is given by the number of FSUs $f_{u}$ required for each user. Furthermore, two variations of this criterion can be made: sorting from least to greatest bandwidth demand (FSU $\uparrow$ ); and ordering from greatest to least slots requirements (FSU $\downarrow$ ).
- Link: The arrangement is given by the number of links $e$ on the shortest route. Likewise, two variations of this order are made: sorting from least to largest amount of links (Link $\uparrow$ ); and, on the other hand, from largest to least amount of links (Link $\downarrow$ ).
- Distance: The sequence is given by the users' path lengths $\ell_{u}$. Also, two variations of this order are done: sorting from shortest to longest path (Distance $\uparrow$ ); and from longest to shortest route (Distance $\downarrow$ ).
In addition, mixed criteria were evaluated. For instance, ordering first by the FSU demands, and the ties are arranged by the number of links in their routes, denoted as FSULink. This way, we tested several mixed criteria such as: Link-FSU, FSU-Distance and Distance-FSU. Remark that, for all the above criteria, all possible ascending and descending combinations were evaluated.

In algorithmic way, to represent the just described subprocedure we write $\mathcal{U}:=\operatorname{Sorting}(\mathcal{U}, \mathcal{L}, \mathcal{R}, \mathcal{F})$.
4) SSA strategy: Solving the spectrum and spatial assignment (SSA) problem involves finding a spectral portion according to the spectral needs for each user in a given core. Remark that this allocation must satisfy the continuity and contiguity constraints.

Most SSA methods use the First-Fit policy to assign spectrum to users, as mentioned in Section II, due to its simplicity and good performance. Therefore, this strategy is used in our SSA strategy proposal.

Algorithm 1 shows the proposed RMLSSA algorithm, including all the sub-procedures explained above.

Lines 2 to 3 refers to the calculation of all users paths $\mathcal{R}$, their modulation formats $\mathcal{M}$, and their corresponding FSU demands $\mathcal{F}$.

In line 4, we execute the Sorting procedure. Remark that the sorting function will proceed depending on the sorting criteria, as explained in Sub-section III-C3.

In lines 5 to 15, we start an iterative procedure searching to assign a subset of FSUs to each user $u$, the so-called SSA algorithm. Then, for each user $u$, we iterate while the slot and core assignment are not complete (line 6).

Note that we use the First-Fit algorithm modified for the EON-SDM architectures. Thus, we start with the first FSU ( $s$ equal to 0 ) on the first core ( $k$ equal to 1 ), in line 7 and 8. Let $s_{u k}^{\max }$ be the maximum value of capacity $c_{e k}$ given for the links $e \in r_{u}$ on the $k$-th core. From lines 9 to 15 we search if there are FSUs available on the $k$-th core on the user path $r_{u}$. These FSUs must comply with continuity and contiguity constraints. Therefore, this search starts from the first FSU to the $s_{u k}^{\max }-f_{u}-1$ slot available, since users demand $f_{u}$ contiguous slots to have successful communication.

Repeating the steps explained above for each set of users $u \in \mathcal{U}$ (lines 5 to 15), we obtain all the users' paths $\mathcal{R}$, modulation formats $\mathcal{M}$, and the updated sets $\mathcal{U}$, and $\mathcal{E}$ with the FSUs assigned to each user.

## IV. Simulation Results

In this section, we illustrate the performance of the RMLSSA proposal by comparing its output with several different criteria for sorting the users on the NSFNet network topology ( 14 nodes and 42 unidirectional links), shown in Figure 2.

The RMLSSA performance was evaluated using an eventdiscrete simulator based on Python. We performed our simulations for different bit-rates, modulation formats, and sorting criteria (Table II).

The network capacity is defined by the frequency spectrum C-band, using an FSU size of 12.5 GHz , leading to a total of 320 FSUs per core. The amount of fiber core used on the multi-core fibers depends on the execution of the methods,

```
Algorithm 1 RMLSSA proposal
    procedure \(\operatorname{RMLSSA}(\mathcal{G}, \mathcal{U}, \mathcal{C})\)
        \(\mathcal{R}:=\operatorname{Routing}(\mathcal{G}, \mathcal{U})\);
        \(\{\mathcal{M}, \mathcal{F}\}:=\operatorname{ML}(\mathcal{L}, \mathcal{U})\);
        \(\mathcal{U}:=\operatorname{Sorting}(\mathcal{U}, \mathcal{L}, \mathcal{R}, \mathcal{F})\);
        for each user \(u \in \mathcal{U}\) do
            while user not assigned do
                \(s:=0\);
                for all \(k \in \mathcal{K}\) do
                    while \(s \leq s_{u k}^{\max }-f_{u}-1\) do
                        if FSUs from \(s\) to \(s+f_{u}-1\) are free in \(k\)-th core of \(r_{u}\) then
                        Assign the slots to user \(u\) in \(k\)-th core of \(r_{u}\);
                            Break;
                            else
                            \(s:=s+1 ;\)
                    \(s:=0 ;\)
        return \(\mathcal{R}, \mathcal{M}, \mathcal{U}, \mathcal{E}\)
```



Fig. 2. NSFNet network.

TABLE II
Simulation Parameters

| Parameter | Value |
| :---: | :---: |
| Each core capacity | 320 slots |
| Bit-rates | $10,40,100,400,1000 \mathrm{Gbps}$ |
| Modulation | BPSK, QPSK, 8-QAM, |
| Format | 16-QAM, 32-QAM, 64-QAM |
| Bandwidth (FSUs) | Table I |

assigning as many cores as needed to allocate all transmission requests.

All network users have different bit-rate demands, which are generated randomly. The procedure chooses between 10 , 40, 100, 400, and 1000 Gbps values. Remark that we use the same seed in order to obtain the same random bit-rate requests. Also, the NSFNet network has several long distance communications. Even more, some of them have more than 4000 km . For these cases, we use the BPSK modulation format.

## A. Performance Metrics

To evaluate the RMLSSA strategies, we use several significant metrics. The most critical ones are related to the network
capacity and spectrum fragmentation. These are explained as follows.

1) Effective Network Capacity: The effective network capacity $\left(\hat{C}_{n e t}\right)$ is defined as the sum of all the effective link capacity of the network link. The effective link capacity is illustrated in Eq. (1). The metric $\hat{C}_{\text {net }}$ has been commonly used to evaluate RSA algorithms [12], [20]. We evaluate $\hat{C}_{n e t}$ as follows:

$$
\begin{equation*}
\hat{C}_{n e t}=\sum_{e \in \mathcal{E}} \hat{c}_{e} . \tag{2}
\end{equation*}
$$

2) Fragmentation: As mentioned in Section II, the presence of non-used FSUs on the network links should be avoided because it constitutes a waste of network resources.

The fragmentation is the proportion of unused spectral slots over the total network capacity. In SDM we can recognize two types of fragmentation: spectrum fragmentation (SpectF) and spatial fragmentation (SpatF). The spectrum fragmentation is the unused FSU in the middle area of the spectrum frequencies resulting from the spectrum assignment. Spatial fragmentation is the unused FSUs located in the last part of each fiber core that cannot serve users due to the limited capacity of each core. We consider that the last fiber core used in each link does not provide spatial fragmentation, since those slots are free to be used by any new user added to the network. The SpectF and SpatF are evaluated as follows (Eq (3) and (4)):

$$
\begin{align*}
\operatorname{SpectF}(\%) & =100 \cdot \frac{\operatorname{SpectF}}{C_{n e t}}  \tag{3}\\
\operatorname{SpatF}(\%) & =100 \cdot \frac{\operatorname{SpatF}}{C_{n e t}} \tag{4}
\end{align*}
$$

3) Not assigned FSUs: We define the not assigned FSUs as the amount of FSUs that remain free after $\hat{c}_{e \hat{k}}$. This free capacity is separately defined since this FSUs can serve new users on the network. The spectrum available is also called Free FSU.

Figure 3 exemplify the spectrum and spatial fragmentation along with the Free FSUs. The figure shows two links for an arbitrary network. Each link is represented as a matrix, where FSUs and cores are represented on the columns and rows, respectively. The FSUs occupied are those marked in gray, computed as follows: (5).

$$
\begin{equation*}
\text { UsedFSU }=\hat{C}_{n e t}-(\text { SpectF }+ \text { SpatF }) \tag{5}
\end{equation*}
$$



Fig. 3. Example of space and spectrum assignment (SSA) on a two links arbitrary network.

In Figure 3, we can see the spectrum fragmentation on link 1 es equal to 6 FSUs $(k=1 \rightarrow \mathrm{FSU}: 4, k=2 \rightarrow \mathrm{FSU}: 1,2$ and $k=3 \rightarrow \mathrm{FSU}: 1,2,3)$, and 4 FSUs in link $2(k=$ $1 \rightarrow \mathrm{FSU}: \emptyset, k=2 \rightarrow \mathrm{FSU}: 2, k=3 \rightarrow \mathrm{FSU}: 1,2,4$ and $k=4 \rightarrow$ FSU : $\emptyset$ ). Consequently the total network fragmentation SpectF is 10 FSUs.

On the other hand, the spatial fragmentation on link 1 is equal to 5 FSUs ( $k=1 \rightarrow \mathrm{FSU}: 8, k=2 \rightarrow \mathrm{FSU}: 5,6,7,8$, $k=3 \rightarrow$ FSU : $\emptyset$ ), and 6 FSUs for link $2(k=1 \rightarrow \emptyset$, $k=2 \rightarrow$ FSU : 5, $6,7,8, k=3 \rightarrow$ FSU : 7, 8 and $k=4 \rightarrow$ FSU : $\emptyset$ ). The total SpatF value is 11 FSUs.

The effective core capacity $c_{e \hat{k}}$ on the link 1 and 2 are given by the last assigned FSU on the last used core $\hat{k}$ ( $c_{1 \hat{3}}=5$ and $c_{2 \hat{4}}=2$ ). Therefore, the effective link capacity on each link given by equation (1) are: $\hat{c}_{1}=16+5=21 \mathrm{FSUs}$ and $\hat{c}_{2}=$ $24+2=26$ FSUs. As a consequence, the effective network capacity given by Eq. (2) is $\hat{\mathcal{C}}_{n e t}=21+26=47$ FSUs.

Finally, the free FSUs (the last available FSU on the last used core) in links 1 and 2 are 3 and 6 FSUs, respectively, composing a network Free FSU value equal to 9 .

## B. Numerical Results

In this work, we perform the RMLSSA method sorting the users based on 15 different tests sorting the users by different criteria, as shown in Table III. This table shows for each test: the last core used; the effective network capacity, the spectrum fragmentation; the spatial fragmentation; and the free FSUs obtained.

The tests were evaluated, distributing the bit-rates randomly to each user, or the worst case possible with all the users demanding 1000 Gbps. Due to the lack of space, the results illustrated here are those obtained in the worst scenario.

The objective is to analyze the different sorting criteria, and its impact on the effective network capacity, and the spectrum and spatial fragmentation metrics.

TABLE III
NUMERICAL RESULTS OBTAINED BY ALL TESTS CONSIDERED IN THIS WORK

| No. | Test | $\hat{k}$ | $\hat{\boldsymbol{C}}_{\text {net }}$ | SpatF | SpectF | Free FSU |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | Random | 5 | 51120 | 12178 | 9632 | 5200 |
| $\mathbf{2}$ | FSU $\downarrow$ | 5 | 47014 | 9858 | 7846 | 6106 |
| $\mathbf{3}$ | FSU $\uparrow$ | 5 | 51440 | 14414 | 7716 | 4240 |
| $\mathbf{4}$ | Link $\downarrow$ | 5 | 45390 | 7642 | 8438 | 5810 |
| $\mathbf{5}$ | Link $\uparrow$ | 6 | 53600 | 11572 | 12718 | 5920 |
| $\mathbf{6}$ | Distance $\downarrow$ | 5 | 41654 | 3838 | 8506 | 5706 |
| $\mathbf{7}$ | Distance $\uparrow$ | 5 | 52800 | 13486 | 10004 | 4160 |
| $\mathbf{8}$ | Link $\downarrow$ FSU $\downarrow$ | 5 | 46616 | 8070 | 9236 | 5864 |
| $\mathbf{9}$ | Link $\uparrow$ FSU $\uparrow$ | 6 | 54160 | 12058 | 12792 | 5360 |
| $\mathbf{1 0}$ | Link $\downarrow$ FSU $\uparrow$ | 5 | 44936 | 7998 | 7628 | 5624 |
| $\mathbf{1 1}$ | Link $\uparrow$ FSU $\downarrow$ | 6 | 53760 | 11492 | 12958 | 5760 |
| $\mathbf{1 2}$ | FSU $\downarrow$ Link $\downarrow$ | 5 | 46616 | 8470 | 8836 | 5864 |
| $\mathbf{1 3}$ | FSU $\uparrow$ Link $\uparrow$ | 6 | 54080 | 12592 | 12178 | 5440 |
| $\mathbf{1 4}$ | FSU $\downarrow$ Link $\uparrow$ | 5 | 46054 | 7164 | 9580 | 5786 |
| $\mathbf{1 5}$ | FSU $\uparrow$ Link $\downarrow$ | 5 | 48960 | 10506 | 9144 | 4800 |

Also, Figure 4 shows the effective network capacity $\hat{C}_{n e t}$ obtained by the 15 tests, adding the proportion of FSUs used by any user (UsedFSU), the spatial fragmentation (SpatF) and the spectrum fragmentation (SpectF). Remark that all users' demands were attended; thus, the FSU occupied (UsedFSU) are the same on all tests.


Fig. 4. Effective network capacity obtained for all test considered in this work. For each test, we add the proportion of the trully used FSUs (UsedFSU), and the spectral and spatial fragmentation (SpectF and SpatF).

We can see that the lowest effective network capacity value are the ones obtained by the Tests 6 (Distance $\downarrow$ ) and $\mathbf{1 0}$ (Link $\downarrow$ FSU $\uparrow$ ). On the other hand, the Tests 9 (Link $\uparrow$ FSU $\uparrow$ ) and 13 (FSU $\uparrow$ Link $\uparrow$ ) are the largest ones. Concerning the network fragmentation, the lowest spatial fragmentation are


Fig. 5. Distribution of the FSUs on each cores divided by the modulation format used (BPSK, QPSK, 8QAM, 16QAM and 32QAM), the not assigned FSUs (free FSU), and the spatial and spectral fragmentation (SpatF and SpectF, respectively), for the two best tests performed tests (Test 6 and 10), and the worst ones (Test 1 and 9).
obtained on Tests 6 (Link $\downarrow$ FSU $\uparrow$ ) with $9.21 \%$ and 14 (FSU $\downarrow$ Link $\uparrow$ ) with $15.55 \%$, and the lowest spectrum fragmentation are obtained on Tests $\mathbf{1 0}$ (Link $\downarrow$ FSU $\uparrow$ ) with $16.97 \%$ and 3 (FSU $\uparrow$ ) with $15 \%$. Otherwise, the largest spatial fragmentation values are obtained by Tests 3 (FSU $\uparrow$ ) with $28.02 \%$ and 7 (Distance $\uparrow$ ) with $25.54 \%$, and the largest spectrum fragmentation are found in Tests $\mathbf{1 1}$ (Link $\uparrow$ FSU $\downarrow$ ) with $24.10 \%$ and 9 (Link $\uparrow$ FSU $\uparrow$ ) with $23.61 \%$.

Consequently, considering the previous remarks, it can be seen that the best-executed criteria are $\mathbf{6}$ and $\mathbf{1 0}$ since they demand fewer cores with the lowest spectrum and spatial fragmentation. Based on previous data, we can conclude that the descending order criteria are, starting with the largest obtain better results in the metrics evaluated compared to the random case. In the case of mixed criteria, those with the first decision criteria descending obtain better results.

To present additional information we select four tests, the two best criteria (Test 6 and 10), and the worst two ones (Test $\mathbf{1}$ and 9), and show them in Figure 5. This figure illustrates the distribution of the modulation formats of the users on each fiber core, including both spectrum and spatial fragmentation and the free FSUs.

Since NFSNet is considered as a wide-area network, the most widely used modulation format is BPSK since there are a lot of long-reach connection requests, as shown in the Figure 5.

Figure 5 exemplify that tests Test $\mathbf{6}$ and $\mathbf{1 0}$ distributes the most complex modulation formats in all the cores, and the BPSK formats are served as soon as possible. On the other hand, the tests with worst results (Test 1 and 9) attend the users with a more efficient modulation format in the first cores, and the users demanding communication with BPSK are distributed on all the fiber cores.

In conclusion, we can see that good practice is to attend first the modulation formats with low bandwidth efficiency primarily since they demand a large number of frequency slots to transmit; and to use the highly efficient modulation formats (demanding less FSUs to transmit) to fill the frequency gaps left on the fiber cores. This way, reducing the fragmentation in each core.

## V. Conclusions

In this work, we analyze the impact of sorting the network users previous to the spatial and spectrum assignment subproblem on the network fragmentation. To this end, we solve
the Routing, Modulation Level, Spatial, and Spectrum Assignment (RMLSSA) problem for EON-SDM architectures with static network operation.

We performed our RMLSSA strategy using 15 different sorting criteria. The best tests were Test 6 (Distance $\downarrow$ ), sorting the users by their path distance in kilometers in descending order, and Test 10 (Link $\downarrow$ FSU $\uparrow$ ) arranging the users first by the number of links of their paths in ascending order, and their FSU sorts the ties demands from lowest to highest ones. As a common characteristic, we find that using sorting criteria in a descending order tends to have a better performance in terms of fragmentation, than arranging users in ascending order. Also, on mixed sorting criteria, a good practice would be to use the first criteria in descending order.

As seen in the experimental results, the order in which the users are allocated affects the spatial and spectral fragmentation. Therefore, choosing proper criteria may obtain a more efficient usage of resources on the frequency spectrum and fiber cores.

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